RESEARCH ARTICLE

Cartesian Products over Interval Valued Intuitionistic Fuzzy Sets of Second Type

K. Rajesh[1], R. Srinivasan[2]

[1]Department of Mathematics, Mahendra Engineering College (Autonomous),
Mallasamudram – 637503, Tamilnadu, India
[2]Department of Mathematics, Islamiah College (Autonomous),
Vaniyambadi, Tamilnadu, India

ABSTRACT: The Intuitionistic Fuzzy Set (IFS) based on fuzzy theory, which is of high efficiency to solve the fuzzy problem, has been introduced by Atanassov. Subsequently, he published the research one step further from the IFS to the Interval Valued Intuitionistic Fuzzy Sets (IVIFS). On the basis of fuzzy set (FS), the IFS are a generalization concept. And the IFS are generalized to the IVIFS. In this paper, we introduce the Cartesian products over Interval Valued Intuitionistic Fuzzy Sets of second type and we prove some equality based on the operations, operators and the relation over IVIFSST.

KEYWORDS: Fuzzy Sets, Intuitionistic Fuzzy Sets, Intuitionistic Fuzzy Sets of Second Type, Interval Valued Intuitionistic Fuzzy Sets, Interval Valued Intuitionistic Fuzzy Sets of Second Type

https://doi.org/10.29294/IJASE.8.4.2022.2462-2466

1. INTRODUCTION

In 1983, Atanassov [1] introduced the notion of Intuitionistic Fuzzy Sets (IFS) in which he has taken non-membership grade along with membership grade in defining IFS. In certain situation, the membership value alone may not be sufficient to determine the nature of events and non-membership also may be relevance. Such situations are better handled by the IFS theory developed by Atanassov.

Atanassov and Gargov (1989) [2] has introduced the Interval Valued Intuitionistic Fuzzy Set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non membership function are intervals rather than exact numbers. Atanassov has defined many operators, identities, relations and different types of distance measures over IVIFS.

The present authors further introduced the new extension of IVIFS namely Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) and established some of their properties [3]. The rest of the paper is designed as follows: In Section 2, provides some basic definitions. In Section 3, we have introduced the Cartesian products over Interval Valued Intuitionistic Fuzzy Sets of Second Type and establish some of their properties.

2. PRELIMINARIES

In this section, we give some basic definitions.

Definition 2.1[1] Let X be a nonempty set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the following form.

$$\mathrm{A} = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

Where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \le \mu_A(x) + \nu_A(x) \le 1$$

Definition 2.2 [1] Let a set X be fixed. An Intuitionistic Fuzzy Set of Second Type (IFSST) A in X is defined as an object of the following form.

$$\mathrm{A} = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \le \mu_A{}^2(x) + \nu_A{}^2(x) \le 1$$

Definition 2.3 [2] An Interval Valued Intuitionistic Fuzzy Sets (IVIFS) A in X is given by

$$\mathrm{A} = \{\langle x, \mathrm{M}_A(x), \mathrm{N}_A(x)\rangle \mid x \in X\}$$

**Corresponding Author: rajeshagm@gmail.com*
Received: 15.03.2022 **Accepted:** 27.04.2022 **Published on**: 18.05.2022

where $M_A : X \to [0,1]$, $N_A : X \to [0,1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and the degree of non-membership of the element x in X, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that

$$M_{AU}(x) + N_{AU}(x) \le 1 \ \forall x \in X$$

Definition 2.4 [3] An Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) A in X is given by

$$A = \{\langle x, M_A(x), N_A(x)\rangle \mid x \in X\}$$

where $M_A: X \to [0,1]$, $N_A: X \to [0,1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and the degree of non-membership of the element x in X, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that

$$M^2{}_{AU}(x) + N^2{}_{AU}(x) \le 1 \ \forall\, x \in X.$$

Definition 2.5 [3, 4] For every two IVIFSST A and B, we have the following relations and operations

1. $\bar{A} = \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle \mid x \in X\}$
2. $A \cup B = \{\langle x, [\max(M_{AL}(x), M_{BL}(x)), \max(M_{AU}(x), M_{BU}(x))], [\min(N_{AL}(x), N_{BL}(x)), \min(N_{AU}(x), N_{BU}(x))]\rangle \mid x \in X\}$
3. $A \cap B = \{\langle x, [\min(M_{AL}(x), M_{BL}(x)), \min(M_{AU}(x), M_{BU}(x))], [\max(N_{AL}(x), N_{BL}(x)), \max(N_{AU}(x), N_{BU}(x))]\rangle \mid x \in X\}$
4. $A + B = \{\langle x, [M^2{}_{AL}(x) + M^2{}_{BL}(x) - M^2{}_{AL}(x). M^2{}_{BL}(x),\ M^2{}_{AU}(x) + M^2{}_{BU}(x) - M^2{}_{AU}(x). M^2{}_{BU}(x)], [N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x)N^2{}_{BU}(x)]\rangle \mid x \in X\}$
5. $A. B = \{\langle x, [M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x) + N^2{}_{BL}(x) - N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x) + N^2{}_{BU}(x) - N^2{}_{AU}(x)N^2{}_{BU}(x)]\rangle \mid x \in X\}$

Definition 2.6 [5] For every IVIFSST, we have the following

Necessity operator

$$\Box A = \{\langle x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{1 - M^2{}_{AU}(x)}\right]\rangle \mid x \in X\}$$

Possibility operator

$$\Diamond\, A = \{\langle x, \left[M_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\}$$

Definition 2.7 [6] Let X be a non-empty set and for every IVIFSST A, we have four operators which map an IFS to an IVIFSST as follows

$$*_1A = \{\langle x, M_{AL}(x), N_{AL}(x)\rangle \mid x \in X\},$$
$$*_2A = \{\langle x, M_{AL}(x), N_{AU}(x)\rangle \mid x \in X\},$$
$$*_3A = \{\langle x, M_{AU}(x), N_{AL}(x)\rangle \mid x \in X\},$$
$$*_4A = \{\langle x, M_{AU}(x), N_{AU}(x)\rangle \mid x \in X\}.$$

3. *Cartesian products over interval valued intuitionistic fuzzy sets of second type*

In this section, we define the Cartesian Products over Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) and also we establish some of their properties.

Definition3.1 The five Cartesian products of two IVIFSSTs, A over X and B over Y are define as follows:

(i) The Cartesian product " $\times_1$ "

$$A \times_1 B = \{\langle\langle x, y\rangle, [M^2{}_{AL}(x). M^2{}_{BL}(y), M^2{}_{AU}(x). M^2{}_{BU}(y)], [N^2{}_{AL}(x). N^2{}_{BL}(y), N^2{}_{AU}(x). N^2{}_{BU}(y)]\rangle \mid x \in X, y \in Y\}$$

(ii) The Cartesian product " $\times_2$ "

$$A \times_2 B = \{\langle\langle x, y\rangle, [M^2{}_{AL}(x) + M^2{}_{BL}(y) - M^2{}_{AL}(x). M^2{}_{BL}(y), M^2{}_{AU}(x) + M^2{}_{BU}(y) - M^2{}_{AU}(x). M^2{}_{BU}(y)], [N^2{}_{AL}(x). N^2{}_{BL}(y), N^2{}_{AU}(x). N^2{}_{BU}(y)]\rangle \mid x \in X, y \in Y\}$$

(iii) The Cartesian product " $\times_3$ "

$$A \times_3 B = \{\langle\langle x, y\rangle, [M^2{}_{AL}(x). M^2{}_{BL}(y), M^2{}_{AU}(x). M^2{}_{BU}(y)], [N^2{}_{AL}(x) + N^2{}_{BL}(y) - N^2{}_{AL}(x). N^2{}_{BL}(y), N^2{}_{AU}(x) + N^2{}_{BU}(y) - N^2{}_{AU}(x). N^2{}_{BU}(y)\rangle \mid x \in X, y \in Y\}$$

(iv) The Cartesian product " $\times_4$ "

$$A \times_4 B = \{\langle (x,y), [\min(M_{AL}(x), M_{BL}(y)), \min(M_{AU}(x), M_{BU}(y)], [\max(N_{AL}(x), N_{BL}(y)), \max(N_{AU}(x), N_{BU}(y)]\rangle | x \in X, y \in Y\}$$

(v) The Cartesian product " $\times_5$ "

$$A \times_5 B = \{\langle (x,y), [\max(M_{AL}(x), M_{BL}(y)), \max(M_{AU}(x), M_{BU}(y)], [\min(N_{AL}(x), N_{BL}(y)), \min(N_{AU}(x), N_{BU}(y)]\rangle | x \in X, y \in Y\}$$

Theorem 3.1 Let X be a non-empty set. For every two IVIFSSTs A and B and for $1 \leq i \leq 4$, we have the following

(i) $*_i (A \times_1 B) = *_i A \times_1 *_i B$,
(ii) $*_i (A \times_2 B) = *_i A \times_2 *_i B$,
(iii) $*_i (A \times_3 B) = *_i A \times_3 *_i B$.

Proof: (i) First we prove the result for i = 1

Let

$$A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle | x \in X\}$$
$$B = \{\langle x, [M_{BL}(x), M_{BU}(x)], [N_{BL}(x), N_{BU}(x)]\rangle | y \in Y\}$$

then,

$$A \times_1 B = \{\langle (x,y), [M^2{}_{AL}(x).M^2{}_{BL}(y), M^2{}_{AU}(x).M^2{}_{BU}(y)], [N^2{}_{AL}(x).N^2{}_{BL}(y), N^2{}_{AU}(x).N^2{}_{BU}(y)]\rangle | x \in X, y \in Y\}$$

Apply the operator $*_1$ in $A \times_1 B$ we have

$$*_1 (A \times_1 B) = \{\langle (x,y), [M^2{}_{AL}(x).M^2{}_{BL}(y)], [N^2{}_{AL}(x).N^2{}_{BL}(y)]\rangle | x \in X, y \in Y\} \quad \text{........ (1)}$$

by the definition

$$*_1 A = \{\langle x, M_{AL}(x), N_{AL}(x)\rangle | x \in X\} \text{ and}$$
$$*_1 B = \{\langle y, M_{BL}(y), N_{BL}(y)\rangle | y \in Y\}$$

then

$$*_1 A \times_1 *_1 B = \{\langle (x,y), [M^2{}_{AL}(x).M^2{}_{BL}(y)], [N^2{}_{AL}(x).N^2{}_{BL}(y)]\rangle | x \in X, y \in Y\} \quad \text{........ (2)}$$

from equation (1) and (2) we have

$$*_1 (A \times_1 B) = *_1 A \times_1 *_1 B$$

Similarly for $i = 2,3$ & 4 we have

$$*_2 (A \times_1 B) = *_2 A \times_1 *_2 B,$$
$$*_3 (A \times_1 B) = *_3 A \times_1 *_3 B,$$
$$*_4 (A \times_1 B) = *_4 A \times_1 *_4 B.$$

(ii) First we prove the result for i = 2

$$A \times_2 B = \{\langle (x,y), [M^2{}_{AL}(x) + M^2{}_{BL}(y) - M^2{}_{AL}(x).M^2{}_{BL}(y), M^2{}_{AU}(x) + M^2{}_{BU}(y) - M^2{}_{AU}(x).M^2{}_{BU}(y)], [N^2{}_{AL}(x).N^2{}_{BL}(y), N^2{}_{AU}(x).N^2{}_{BU}(y)]\rangle | x \in X, y \in Y\}$$

Apply the operator $*_2$ in $A \times_2 B$ we have

$$*_2 (A \times_2 B) = \{\langle (x,y), [M^2{}_{AL}(x) + M^2{}_{BL}(y) - M^2{}_{AL}(x).M^2{}_{BL}(y)], [N^2{}_{AU}(x).N^2{}_{BU}(y)]\rangle | x \in X, y \in Y\} \text{......(3)}$$

by the definition

$$*_2 A = \{\langle x, M_{AL}(x), N_{AU}(x)\rangle | x \in X\} \text{ and}$$
$$*_2 B = \{\langle y, M_{BL}(y), N_{BU}(y)\rangle | y \in Y\}$$

then

$$*_2 A \times_2 *_2 B = \{\langle (x,y), [M^2{}_{AL}(x) + M^2{}_{BL}(y) - M^2{}_{AL}(x).M^2{}_{BL}(y)], [N^2{}_{AU}(x).N^2{}_{BU}(y)]\rangle | x \in X, y \in Y\} \text{......(4)}$$

from equation (3) and (4) we have

$$*_2 (A \times_2 B) = *_2 A \times_2 *_2 B$$

Similarly for $i = 1,3$ & 4 we have

$$*_1 (A \times_2 B) = *_1 A \times_2 *_1 B,$$
$$*_3 (A \times_2 B) = *_3 A \times_2 *_3 B,$$
$$*_4 (A \times_2 B) = *_4 A \times_2 *_4 B.$$

(iii) First we prove the result for i = 3

$$A \times_3 B = \{\langle\langle x,y\rangle, [M^2{}_{AL}(x).M^2{}_{BL}(y), M^2{}_{AU}(x).M^2{}_{BU}(y)], [N^2{}_{AL}(x)+N^2{}_{BL}(y)- N^2{}_{AL}(x).N^2{}_{BL}(y), N^2{}_{AU}(x)+N^2{}_{BU}(y)-N^2{}_{AU}(x).N^2{}_{BU}(y)\rangle | x\in X, y\in Y\}$$

Apply the operator $*_3$ in $A \times_3 B$ we have

$$*_3\,(A\times_3 B) = \{\langle\langle x,y\rangle, [M^2{}_{AU}(x).M^2{}_{BU}(y)], [N^2{}_{AL}(x)+N^2{}_{BL}(y)-N^2{}_{AL}(x).N^2{}_{BL}(y)]\rangle | x\in X, y\in Y\} \ldots\ldots\ldots(5)$$

by the definition

$$*_3A = \{\langle x, M_{AU}(x), N_{AL}(x)\rangle | x\in X\} \text{ and}$$
$$*_3B = \{\langle y, M_{BU}(y), N_{BL}(y)\rangle | y\in Y\}$$

then

$$*_3\,A\times_3 *_3\,B = \{\langle\langle x,y\rangle, [M^2{}_{AU}(x).M^2{}_{BU}(y)], [N^2{}_{AL}(x)+N^2{}_{BL}(y)-N^2{}_{AL}(x).N^2{}_{BL}(y)]\rangle | x\in X, y\in Y\} \ldots\ldots\ldots\ldots(6)$$

from equation (5) and (6) we have

$$*_3\,(A\times_3 B) = *_3\,A\times_3 *_3\,B$$

Similarly for $i = 1,2$ & 4 we have

$$*_1\,(A\times_3 B) = *_1\,A\times_3 *_1\,B,$$
$$*_2\,(A\times_3 B) = *_2\,A\times_3 *_2\,B,$$
$$*_4\,(A\times_3 B) = *_4\,A\times_3 *_4\,B.$$

Theorem 3.2 Let X be a non-empty set. For every two IVIFSSTs A and B and for $1 \le i \le 4$, we have the following

(i) $*_i\,(A\times_4 B) = *_i\,A\ \times_4 *_i\,B,$

(ii) $*_i\,(A\times_5 B) = *_i\,A\ \times_5 *_i\,B.$

Proof: (i) First we prove the result for i = 4

Let

$$A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle | x\in X\}$$
$$B = \{\langle x, [M_{BL}(x), M_{BU}(x)], [N_{BL}(x), N_{BU}(x)]\rangle | y\in Y\}$$

then,

$$A\times_4 B = \{\langle x,y\rangle, [\min(M_{AL}(x), M_{BL}(y)), \min(M_{AU}(x), M_{BU}(y)], [\max(N_{AL}(x), N_{BL}(y)), \max(N_{AU}(x), N_{BU}(y)]\rangle | x\in X, y\in Y\}$$

Apply the operator $*_4$ in $A\times_4 B$ we have

$$*_4\,(A\times_4 B) = \{\langle x,y\rangle, \min(M_{AU}(x), M_{BU}(y), \max(N_{AU}(x), N_{BU}(y)\rangle | x\in X, y\in Y\} \ldots\ldots\ldots\ldots (7)$$

by the definition

$$*_4A = \{\langle x, M_{AU}(x), N_{AU}(x)\rangle | x\in X\} \text{ and}$$
$$*_4B = \{\langle y, M_{BU}(y), N_{BU}(y)\rangle | y\in Y\}$$

Then

$$*_4\,A\times_4 *_4\,B = \{\langle x,y\rangle, \min(M_{AU}(x), M_{BU}(y), \max(N_{AU}(x), N_{BU}(y)\rangle | x\in X, y\in Y\} \ldots\ldots\ldots\ldots\ldots(8)$$

from equation (7) and (8) we have

$$*_4\,(A\times_4 B) = *_4\,A\times_4 *_4\,B$$

Similarly for $i = 1, 2$ & 3 we have

$$*_1\,(A\times_4 B) = *_1\,A\times_4 *_1\,B,$$
$$*_2\,(A\times_4 B) = *_2\,A\times_4 *_2\,B$$
$$*_3\,(A\times_4 B) = *_3\,A\times_4 *_3\,B$$

(ii) First we prove the result for i = 5

$$A\times_5 B = \{\langle x,y\rangle, [\max(M_{AL}(x), M_{BL}(y)), \max(M_{AU}(x), M_{BU}(y)], [\min(N_{AL}(x), N_{BL}(y)), \min(N_{AU}(x), N_{BU}(y)]\rangle | x\in X, y\in Y\}$$

Apply the operator $*_1$ in $A\times_5 B$ we have

$$*_1\,(A\times_5 B) = \{\langle x,y\rangle, [\max(M_{AL}(x), M_{BL}(y))], [\min(N_{AL}(x), N_{BL}(y)]\rangle | x\in X, y\in Y\} \ldots\ldots(9)$$

by the definition

$$*_1A = \{\langle x, M_{AL}(x), N_{AL}(x)\rangle | x\in X\} \text{ and}$$
$$*_1B = \{\langle y, M_{BL}(y), N_{BL}(y)\rangle | y\in Y\}$$

then

$*_1 A \times_5 *_1 B = \{\langle (x, y), [\max(M_{AL}(x), M_{BL}(y))], [\min(N_{AL}(x), N_{BL}(y)]\rangle | x \in X, y \in Y\}$....(10)
from equation (9) and (10) we have

$$*_1 (A \times_5 B) = *_1 A \times_5 *_1 B$$

Similarly for $i = 2,3$ & 4 we have

$$*_2 (A \times_5 B) = *_2 A \times_5 *_2 B,$$
$$*_3 (A \times_5 B) = *_3 A \times_5 *_3 B,$$
$$*_4 (A \times_5 B) = *_4 A \times_5 *_4 B.$$

CONCLUSION

We have introduced the Cartesian products over Interval Valued Intuitionistic Fuzzy Sets of Second Type also we established some of their properties. It is still open to define some more operators over IVIFSST.

REFERENCES

[1] Andonov, V. 2008, On some properties of one Cartesian product over intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*, 14(1), 12-19.

[2] Atanassov K. T. 1986, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, 87-96.

[3] Atanassov K. T., Gargov, G. 1989, Interval Valued Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems 31, 343-349.

[4] Atanassov K. T. 1999, Intuitionistic fuzzy sets: Theory and Applications, Springer Verlag, Heidelberg.

[5] Rajesh K., Srinivasan R. 2017, Interval Valued Intuitionistic Fuzzy Sets of Second Type, Advances in Fuzzy Mathematics, 12(4), 845-853.

[6] Rajesh K., Srinivasan R. 2017, Some Operators on Interval Valued Intuitionistic Fuzzy Sets of Second Type, International Journal of Scientific Research and Management, 5(8), 6685-6688.

[7] Rajesh K., Srinivasan R. 2017, Operators on Interval Valued Intuitionistic Fuzzy Sets of Second Type, International Journal of Mathematical Archive, 8(8), 35-40.

[8] Rajesh K., Srinivasan R. 2018, A Study on Interval Valued Intuitionistic Fuzzy Sets of Second Type and their Operators, International Journal of Current Advanced Research, 7(1), 66-68.

[9] Rajesh K., Srinivasan R. 2017, A Study on Some Operators over Interval Valued Intuitionistic Fuzzy Sets of Second Type, International Journal of Innovative Research in Science, Engineering and Technology, 6(11), 21051-21057.

[10] Rajesh K., Srinivasan R. 2017, Modal Type Operators over Interval Valued Intuitionistic Fuzzy Sets of Second Type, International Journal of Fuzzy Mathematical Archive, 14(2), 253-260.

[11] Varghese, A., Kuriakose. S. 2012.Cartesian product over intuitionistic fuzzy sets. International Journal of Fuzzy Sets, 2(1): 21 – 27.

[12] Varghese, A., Kuriakose S.2012. More on Cartesian products over intuitionistic fuzzy sets. International Mathematical Forum, 2012, 7(23): 1129-1133.

[13] Zadeh L. A. 1965, Fuzzy sets, Inform and Control, 8, 338-353.

[14] Zeng, D. S., Wu Z. Z. 2004, Cartesian product and correlation of interval valued intuitionistic fuzzy sets. *Journal of Chengdu University of Information Technology*, 19(3): 458-462.

[15] Zhimming Zhang. 2013, Interval Valued Intuitionistic Hesitancy Fuzzy Aggregation Operators and their application in group decision making. Journals of Applied Mathematics. 33, 1-25.